CÁLCULO DE ÁREAS

1.- Calcular el área del recinto determinado por la función $f(x)=x^2-3x+2$, el eje OX y las rectas $x=0$ y $x=3$. Sol: 11/6

2.- Area del recinto limitado por la curva: $y= 1/((x+1)(x+3))$ entre $x=0$ y $x=1$. Sol: 1/2 ln(3/2)

3.- Area del recinto limitado por la curva: $y = \ln(x+3)$, el eje OX, entre $x=0$ y $x=1$. Sol: 4 ln4 - 3 ln3 - 1

4.- Area del recinto limitado por la gráfica de la función: $f(x)=\text{sen}(x/2)$ y el eje OX desde $x=0$ hasta $x=\pi$. Sol: 2

5.- Area del recinto limitado por las funciones: $f(x)=4x-x^2$ y $g(x)=x^2+2x$. Sol: 1/3

6.- Area comprendida entre la función: $f(x)=x^3-4x^2+3x$ y el eje OX. Sol: 37/12

7.- Area del recinto limitado por la gráfica de $f(x)=\cos x$, el eje OX y las rectas $x=0$ y $x=\pi$. Sol: 2

8.- Area del recinto acotado del plano, limitado por la gráfica de $f(x)=x^2/(1+x^2)$, el eje OX y las recta $x=-1$ y $x=1$. Nota: $tg(-\pi/4) = -1$; $tg(\pi/4) = 1$ Sol: $2-\pi/2$

9.- Calcular el valor de "m" para que el área del recinto limitado por la curva $y=x^2$ y la recta $y=mx$ sea 9/2 u.a. Sol: ± 3

10.- Area limitada por $f(x)=xe^{-x}$, el eje OY y la ordenada en el máximo. Sol: 3/e-1.

11.- Obtener el área comprendida entre la función $y=e^x$ y la tangente a la curva en $x=1$. y el eje Y. Sol: e/2 - 1

12.- Area del recinto limitado por la curva $y=xe^x$, el eje OY y la ordenada correspondiente al punto mínimo de la curva. Sol: 3/e-1

13.- Area limitada por las curvas: $y=-x^2-2x+3$ y la recta $y=3$. Sol: 4/3

14.- Area de la región del plano delimitada por los ejes de coordenadas y la gráfica de la función $f(x)=(x-1)e^{-x}$. Sol: 1/e

15.- Hallar el área de la región del plano limitada por la curva $y = (x-1)\, e^{-x}$, el eje de abscisas desde el punto de corte hasta la abscisa en el máximo. Sol: $1/e-2/e^2$

16.- Hallar el área de la región del plano limitada por las curvas $y = \ln x$, $y = 3$ y los ejes de coordenadas. Sol: e^3-1

17.- Hallar el área comprendida entre la curva $y = \ln x$ desde el punto de corte con el eje OX hasta el punto de abscisa $x = e$. Sol: 1

18.- Hallar el valor de "a" para que el área de la región limitada por la curva $y = -x^2+a$ y el eje OX sea igual a 36. Sol: $a = 9$

19.- Calcular el área de las regiones del plano limitadas por las curvas:
a) $y = x^2-3x$ y el eje OX
b) $y = |x^2-5x+4|$ y el eje OX
c) $y = x(x-1)(x-3)$ y el eje OX

d) $y = x^3-6x^2+8x$ y el eje OX
Sol: a) 9/2; b) 9/2; c) 37/12; d) 8

20.- Calcular el área comprendida entre la función $y=\ln x$, el eje OX y la tangente a la función en el punto $x=e$. Sol: $e/2 -1$

21.- Halla el área determinada por las curvas $y=x^2$, $y=1/x$ y la recta $x=2$. Sol: $7/3 - \ln 2$

22.- Halla el área determinada por $y=x^2+1$, su recta tangente en $x=1$ y el eje OY. Sol: 1/3

23.- Halla el área determinada por $y=x^2+1$, su recta normal en $x=1$ y los ejes. Sol: 16/3

24.- Halla el área comprendida entre las curvas $y=x$, $y=1/x$, $y=-7/8\, x + 15/4$, siendo $x \geq 1$. Sol: $15/4-\ln 4$

25.- Halla el área encerrada entre las curvas $y=x^4-4x^2$, $y=x^2-4$. Sol: 8

26.- Halla el área comprendida entre las curvas $y=x^3-x$, $y=3x$. Sol: 8

27.- Halla el área comprendida entre las gráficas de la curvas: $y=-x^4+2x^2$ e $y=1$. Sol: 16/15

28.- Area comprendida entre $y=x^3-x^2$ y el eje OX. Sol: 1/12

29.- Area comprendida entre la curva $y=x/(x^2-5x+4)$ y las rectas $x=5$ y $x=7$. Sol: $4/3 \ln 3 + 1/3 \ln 4 - 1/3 \ln 6$

30.- Area encerrada entre la curva $x^2/(2x-2)$ y las rectas $x=3$ e $y=2$. Sol: $7/4 + 1/2 \ln 2$.

31.- Area comprendida entre la curva $y=\ln(x^2+1)$ y la curva $y=\ln 5$. Nota: $\mathrm{arctg}(-\alpha)=-\mathrm{arctg}(\alpha)$. Sol: $8 - 4\, \mathrm{arctg}(2)$

32.- Area comprendida entre la curva $y=|x-1|$ e $y=2$. Sol: 4

33.- Halla el área comprendida entre la gráfica de las funciones: $y=-x^2+2x$ e $y=x^3(x-2)$. Sol: 44/15

34.- Halla el área comprendida entre la gráfica de las funciones: $y=x^2-2x$ e $y=x^3(x-2)$. Sol: 49/30

35.- Halla el área comprendida entre la gráfica de las funciones: $y=-x^4+2x^2$, $y=x+2$ e $y=-x+2$. Sol: 31/15

36.- Halla el área comprendida entre la gráfica de la función $y=\mathrm{tg}(x)$, el eje OX y la recta $x=\pi/4$. Sol: $\ln(\sqrt{2})$

37.- Halla el área comprendida entre la gráfica de las funciones: $y=2-x^2$ e $y=|x|$. Sol: 7/3

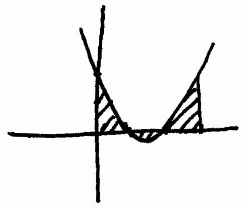
38.- Halla el área determinada por las curvas $y=x^2$, $y=1/x$ y la recta $y=2$.
Sol: $4\sqrt{2}/3-2/3+\ln(1/2)$

① $f(x) = x^2 - 3x + 2$

$x = 0$

$x = 3$

$x = \frac{3 \pm \sqrt{9 - 4 \cdot 2}}{2} = \frac{3 \pm 1}{2} < \begin{matrix} x = 2 \\ x = 1 \end{matrix}$



$$A = \int_0^1 (x^2 - 3x + 2)\,dx - \int_1^2 (x^2 - 3x + 2)\,dx + \int_2^3 (x^2 - 3x + 2)\,dx =$$

$$= \frac{x^3}{3} - 3\frac{x^2}{2} + 2x \Big]_0^1 - \frac{x^3}{3} - 3\frac{x^2}{2} + 2x \Big]_1^2 + \frac{x^3}{3} - 3\frac{x^2}{2} + 2x \Big]_2^3 =$$

$$= \frac{1}{3} - \frac{3}{2} + 2 - 0 - \left(\frac{8}{3} - 3 \cdot \frac{4}{2} + 2 \cdot 2 - \frac{1}{3} + \frac{3}{2} - 2\right) + \frac{3^3}{3} - 3\frac{3^2}{2} + 2 \cdot 3 - \frac{8}{3} + 3 \cdot \frac{4}{2} + 2 \cdot 2 =$$

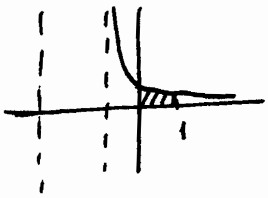
$$= \boxed{\frac{59}{6}}$$

---

② $y = \frac{1}{(x+1)(x+3)}$

$x = 0$

$x = 1$



$$A = \int_0^1 \frac{dx}{(x+1)(x+3)} =$$

$$\frac{1}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3} = \frac{A(x+3) + B(x+1)}{(x+1)(x+3)} \Rightarrow 1 = A(x+3) + B(x+1)$$

$x = -1 \rightarrow 1 = 2A \rightarrow A = 1/2$

$x = -3 \rightarrow 1 = -2B \rightarrow B = -1/2$

$$A = \int_0^1 \left(\frac{1/2}{x+1} - \frac{1/2}{x+3}\right) dx = \frac{1}{2}\ln|x+1| - \frac{1}{2}\ln|x+3| \Big]_0^1 = \frac{1}{2}\ln 2 - \frac{1}{2}\ln 4 - \frac{1}{2}\ln 1 +$$

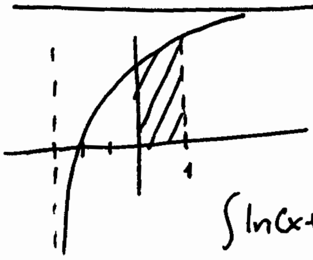
$$+ \frac{1}{2}\ln 3 = \frac{1}{2}(\ln 2 - \ln 4 + \ln 3) = \frac{1}{2}\ln\frac{2 \cdot 3}{4} = \frac{1}{2}\ln 3/2 = \boxed{\ln\sqrt{3/2}}$$

---

③ $y = \ln(x+3)$

$x = 0$

$x = 1$



$$A = \int_0^1 \ln(x+3)\,dx =$$

$$\int \ln(x+3)\,dx = x\ln(x+3) - \int \frac{x}{x+3}\,dx = x\ln(x+3) - \int 1 - \frac{3}{x+3}$$

$u = \ln(x+3)$ $\quad du = \frac{dx}{x+3}$

$dv = dx$ $\quad v = x$

$$\begin{array}{r|l} x & x+3 \\ \hline -x-3 & 1 \\ \cancel{x} -3 & \end{array}$$

$$A = x\ln|x+3| - x + 3\ln|x+3| \Big]_0^1 = (x+3)\ln(x+3) - x \Big]_0^1 = 4 \cdot \ln 4 - 1 - 3\ln 3 + 0 =$$

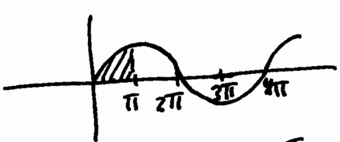
$$= \boxed{4\ln 4 - 3\ln 3 - 1}$$

---

④ $f(x) = \text{sen}(x/2)$

$x = 0$

$x = \pi$



$$A = \int_0^\pi \text{sen}(x/2)\,dx = 2\int_0^\pi \text{sen}(x/2)\,\frac{1}{2}\,dx =$$

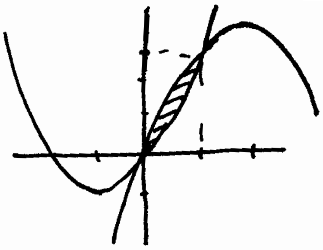
$$= 2\left(-\cos(x/2)\right]_0^\pi = 2\left(-\cos\pi/2 + \cos 0\right) = 2(0 + 1) = \boxed{2}$$

⑤ $\left.\begin{array}{l} f(x) = 4x - x^2 \\ g(x) = x^2 + 2x \end{array}\right\}$ $\left.\begin{array}{l} y = 4x - x^2 \\ y = x^2 + 2x \end{array}\right\}$ $4x - x^2 = x^2 + 2x$

$2x^2 - 2x = 0$

$x^2 - x = 0$

$x(x-1) = 0 \begin{cases} x = 0 \\ x = 1 \end{cases}$



$$A = \int_0^1 \left(4x - x^2 - (x^2 + 2x)\right) dx = \int_0^1 (2x - 2x^2)\,dx = x^2 - 2\frac{x^3}{3}\Big]_0^1 = 1 - \frac{2}{3} - 0 = \boxed{1/3}$$

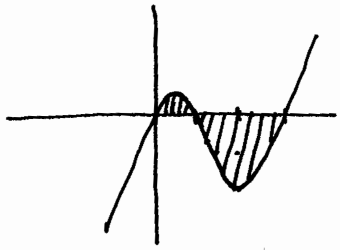
---

⑥ $f(x) = x^3 - 4x^2 + 3x$

OX

$x^3 - 4x^2 + 3x = 0$

$x(x^2 - 4x + 3) = 0 \begin{cases} x = 0 \\ x = 1 \\ x = 3 \end{cases}$



$$A = \int_0^1 (x^3 - 4x^2 + 3x)\,dx - \int_1^3 (x^3 - 4x^2 + 3x)\,dx =$$

$$= \frac{x^4}{4} - 4\frac{x^3}{3} + 3\frac{x^2}{2}\Big]_0^1 - \left(\frac{x^4}{4} - 4\frac{x^3}{3} + 3\frac{x^2}{2}\right]_1^3 = \frac{1}{4} - \frac{4}{3} + \frac{3}{2} - 0 - \left(\frac{81}{4} - 4\frac{27}{3} + 3\frac{9}{2} - \frac{1}{4} + \frac{4}{3} - \frac{3}{2}\right)$$

$$= \frac{1}{4} - \frac{4}{3} + \frac{3}{2} - \frac{81}{4} + 36 - \frac{27}{2} + \frac{1}{4} - \frac{4}{3} + \frac{3}{2} = \boxed{\frac{37}{12}}$$

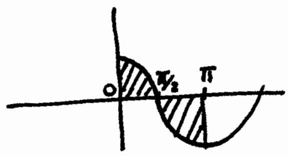
---

⑦ $f(x) = \cos x$

OX

$x = 0$

$x = \pi$



$$A = \int_0^{\pi/2} \cos x\,dx - \int_{\pi/2}^{\pi} \cos x\,dx =$$

$$A = \operatorname{sen} x\Big]_0^{\pi/2} - \operatorname{sen} x\Big]_{\pi/2}^{\pi} = \operatorname{sen}\pi/2 - \operatorname{sen} 0 - \operatorname{sen}\pi + \operatorname{sen}\pi/2 =$$

$$= 1 - 0 - 0 + 1 = \boxed{2}$$

---

⑧ $f(x) = \dfrac{x^2}{1+x^2}$

OX

$x = -1$

$x = 1$

$\dfrac{x^2}{1+x^2} = 0 \Rightarrow x = 0$

$$A = \left|\int_{-1}^{0} \frac{x^2}{1+x^2}\,dx\right| + \left|\int_0^1 \frac{x^2}{1+x^2}\,dx\right| = \left|\int_{-1}^{0}\left(1 - \frac{1}{1+x^2}\right)dx\right| + \left|\int_0^1 \left(1 - \frac{1}{1+x^2}\right)dx\right| =$$

$$\begin{array}{r|l} x^2 & x^2+1 \\ \hline -x^2-1 & 1 \\ \hline / \ -1 & \end{array}$$

$$= \left|x - \operatorname{arctg} x\Big]_{-1}^{0}\right| + \left|x - \operatorname{arctg} x\Big]_0^1\right| =$$

$$= \left|0 - \operatorname{arctg} 0 - (-1) + \operatorname{arctg}(-1)\right| + \left|1 - \operatorname{arctg} 1 - 0 + \operatorname{arctg} 0\right| =$$

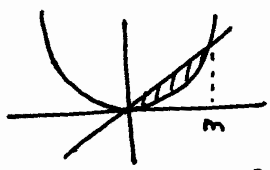
$$= \left|1 - \frac{\pi}{4}\right| + \left|1 - \frac{\pi}{4}\right| = 2\left(1 - \frac{\pi}{4}\right) = \boxed{2 - \frac{\pi}{2}}$$

---

⑨ $\left.\begin{array}{l} y = x^2 \\ y = mx \\ A = \frac{9}{2} \end{array}\right\}$ $x^2 = mx$

$x^2 - mx = 0$

$x(x-m) = 0 \begin{cases} x = 0 \\ x = m \end{cases}$



$$A = \left|\int_0^m (x^2 - mx)\,dx\right| = \left|\frac{x^3}{3} - m\frac{x^2}{2}\Big]_0^m\right| = \left|\frac{m^3}{3} - m\frac{m^2}{2} - 0\right| = \left|\frac{m^3}{3} - \frac{m^3}{2}\right| = \frac{m^3}{6}$$

$$\frac{m^3}{6} = \frac{9}{2} \Rightarrow m^3 = \frac{9 \cdot 6}{2} = 27 \Rightarrow m = \sqrt[3]{27} \Rightarrow \boxed{m = 3}$$

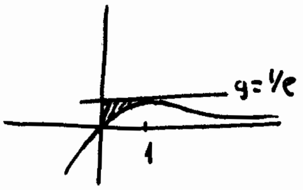
10) $f(x) = x \cdot e^{-x}$

OY

ordenada en el máximo

$$f'(x) = e^{-x} + x \cdot e^{-x}(-1) = e^{-x}(1-x) = 0 \begin{cases} e^{-x} \neq 0 \\ 1-x=0 \Rightarrow x=1 \end{cases}$$

$$f(1) = 1 \cdot e^{-1} = 1/e$$



$$A = \int_0^1 \left(\frac{1}{e} - xe^{-x}\right) dx = \frac{1}{e}x + (x+1)e^{-x} \Big]_0^1 = \frac{1}{e} + \frac{2}{e} - (0 + 1 \cdot e^0) =$$

$$= \boxed{\frac{3}{e} - 1}$$

$$\int -x e^{-x} dx = x e^{-x} - \int e^{-x} dx = x e^{-x} + e^{-x} = (x+1)e^{-x}$$

$u = x \qquad du = dx$

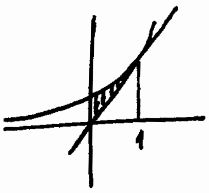
$dv = -e^{-x}dx \qquad v = e^{-x}$

---

11) $y = e^x$

tgte $x=1$

OY



Tangente: $y - y_0 = m(x - x_0) \Rightarrow y - f(1) = f'(1)(x-1)$

$f'(x) = e^x \Rightarrow f'(1) = e$

$f(1) = e^1 = e$

$y - e = e(x-1)$

$y - e = ex - e$

$y = ex$

$$A = \int_0^1 (e^x - ex) dx = e^x - e\frac{x^2}{2} \Big]_0^1 = e - \frac{e}{2} - (e^0 - 0) = \boxed{\frac{e}{2} - 1}$$

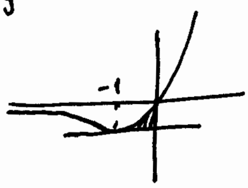
---

12) $y = xe^x$

OY

ordenada mínimo

$$y' = e^x + xe^x = (1+x)e^x = 0 \begin{cases} e^x \neq 0 \\ 1+x=0 \Rightarrow x=-1 \end{cases} \qquad f(-1) = -1e^{-1} = -\frac{1}{e}$$



$$A = \int_{-1}^0 \left(xe^x - \left(-1/e\right)\right) dx = (x-1)e^x + \frac{1}{e}x \Big]_{-1}^0 =$$

$$= -1e^0 + 0 - \left(-2e^{-1} - \frac{1}{e}\right) =$$

$$= -e^0 - \left(-\frac{3}{e}\right) = \boxed{\frac{3}{e} - 1}$$

$$\int x \cdot e^x dx = x e^x - \int e^x dx = x e^x - e^x = (x-1)e^x$$

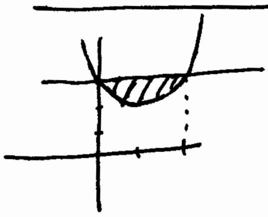
$u = x \qquad du = dx$

$dv = e^x dx \qquad v = e^x$

---

13) $y = x^2 - 2x + 3$

$y = 3$



$$A = \int_0^2 \left[3 - (x^2 - 2x + 3)\right] dx = \int_0^2 (-x^2 + 2x) dx$$

$$= -\frac{x^3}{3} + x^2 \Big]_0^2 = -\frac{8}{3} + 4 - 0 = \boxed{\frac{4}{3}}$$

$x^2 - 2x + 3 = 3$

$x^2 - 2x = 0$

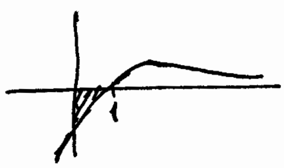
$x(x-2) = 0 \begin{cases} x=0 \\ x=2 \end{cases}$

---

14) $f(x) = (x-1)e^{-x}$

Ejes

$f(x) = 0 \Rightarrow (x-1)e^{-x} = 0 \Rightarrow x = 1$



$$A = -\int_0^1 (x-1)e^{-x} dx$$

$$\int (x-1)e^{-x} dx = -(x-1)e^{-x} - \int -e^{-x} dx = -(x-1)e^{-x} - e^{-x}$$

$u = x-1 \qquad du = dx$

$dv = e^{-x}dx \qquad v = -e^{-x}$

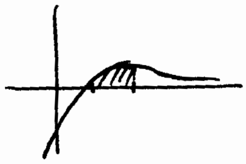
$$A = -\left[-(x-1)e^{-x} - e^{-x}\right]_0^1 = x e^{-x} \Big]_0^1 = 1e^{-1} - 0 = \frac{1}{e}$$

15) $y=(x-1)e^{-x}$
Corte
Máximo

$y'=e^{-x}-(x-1)e^{-x}=(2-x)e^{-x}=0$

$x=2$

$A=\int_1^2 (x-1)e^{-x}\,dx = -xe^{-x}\Big]_1^2 = -2e^{-2}+e^{-1}$



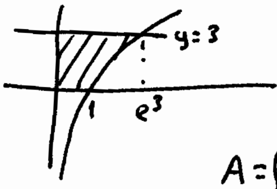
$\int (x-1)e^{-x}\,dx = -(x-1)e^{-x} - \int -e^{-x} = -(x-1)e^{-x} - e^{-x} = -xe^{-x}$

$u=x-1 \quad du=dx$

$dv=e^{-x} \quad v=-e^{-x}$

$A=\boxed{-\frac{2}{e^2}+\frac{1}{e}}$

16) $y=\ln x$
$y=3$
ejes



$\ln x = 3$
$x=e^3$

$A=\int_0^1 3\,dx + \int_1^{e^3}(3-\ln x)\,dx = 3x\Big]_0^1 + 3x-(x\ln x - x)\Big]_1^{e^3} =$

$\int \ln x\,dx = x\ln x - \int x\frac{dx}{x} = x\ln x - x$

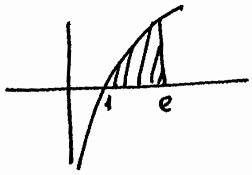
$u=\ln x \quad du=\frac{dx}{x}$

$dv=dx \quad v=x$

$= 3-0+3e^3-e^3\ln e^3+e^3-3+\ln 1-1 =$

$= 3+4e^3-3e^3\underbrace{\ln e}_{1}-4+\underbrace{\ln 1}_{0} = \boxed{e^3-1}$

17) $y=-x^2+a$
Corte OX
$x=e$

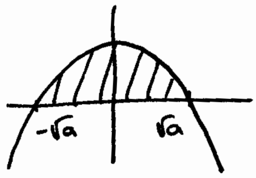


$A=\int_1^e \ln x\,dx = x\ln x - x\Big]_1^e =$ Mirar ej 16

$= e\ln e - e - 1\ln 1 + 1 = e-e-0+1 = \boxed{1}$

18) $y=-x^2+a$, OX, $A=36$

$-x^2+a=0$
$x=\pm\sqrt{a}$



$A=36=\int_{-\sqrt{a}}^{\sqrt{a}}(-x^2+a)\,dx = -\frac{x^3}{3}+ax\Big]_{-\sqrt{a}}^{\sqrt{a}}$

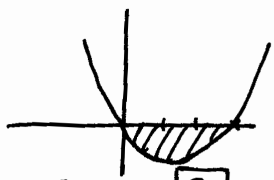
$= -\frac{\sqrt{a}^3}{3}+a\sqrt{a}+\frac{(-\sqrt{a})^3}{3}-a(-\sqrt{a}) = -\frac{a\sqrt{a}}{3}+a\sqrt{a}-\frac{a\sqrt{a}}{3}+a\sqrt{a} = \left(2-\frac{2}{3}\right)a\sqrt{a}$

$\frac{4.a\sqrt{a}}{3}=36 \Rightarrow (\sqrt{a})^3=\frac{36.3}{4}=27$

$(\sqrt{a})^3=27 \Rightarrow \sqrt{a}=\sqrt[3]{27} \Rightarrow \sqrt{a}=3 \Rightarrow \boxed{a=9}$

19) a) $y=x^2-3x$, eje OX

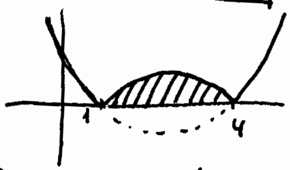
$x^2-3x=0$
$x(x-3)=0 \begin{cases} x=0 \\ x=3 \end{cases}$



$A=-\int_0^3 (x^2-3x)\,dx = -\left[\frac{x^3}{3}-3\frac{x^2}{2}\right]_0^3 = -\frac{27}{3}+\frac{27}{2}-0 = \boxed{\frac{9}{2}}$

b) $y=|x^2-5x+4|$
eje OX

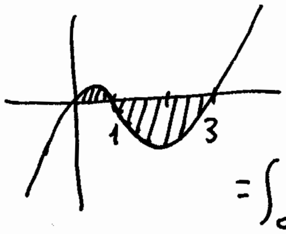
$x^2-5x+4=0 \begin{cases} x=1 \\ x=4 \end{cases}$



$A=\left|\int_1^4 (x^2-5x+4)\,dx\right| = \left|\frac{x^3}{3}-5\frac{x^2}{2}+4x\Big]_1^4\right| = \left|\frac{64}{3}-40+16-\left(\frac{1}{3}-\frac{5}{2}+4\right)\right| =$

$=\boxed{\frac{9}{2}}$

c) $y = x(x-1)(x-3)$
eje OX

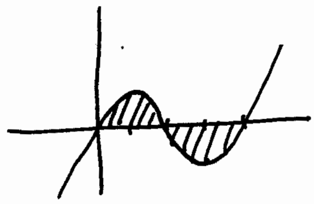


$$A=\int_0^1 (x^2-x)(x-3)\,dx - \int_1^3 (x^2-x)(x-3)\,dx$$
$$= \int_0^1 (x^3-4x^2+3x)\,dx - \int_1^3 (x^3-4x^2+3x)\,dx =$$
$$= \frac{x^4}{4} - \frac{4x^3}{3} + 3\frac{x^2}{2}\Big]_0^1 - \left(\frac{x^4}{4} - \frac{4x^3}{3} + 3\frac{x^2}{2}\Big]_1^3 \right. =$$
$$= \frac{1}{4} - \frac{4}{3} + \frac{3}{2} - 0 - \left(\frac{81}{4} - \frac{4 \cdot 27}{3} + \frac{27}{2} - \frac{1}{4} + \frac{4}{3} - \frac{3}{2} \right) = \boxed{\frac{37}{12}}$$

---

d) $y = x^3 - 6x^2 + 8x$
· eje OX

$x^3 - 6x^2 + 8x = 0$
$x(x^2-6x+8) = 0 \quad x = 0, \; x = 4, \; x = 2$



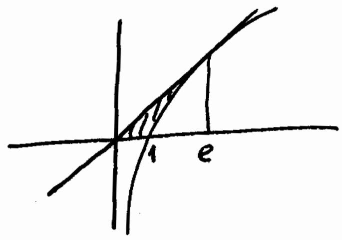
$$A = \int_0^2 (x^3-6x^2+8x)\,dx - \int_2^4 (x^3-6x^2+8x)\,dx =$$
$$= \frac{x^4}{4} - \frac{6x^3}{3} + 8\frac{x^2}{2}\Big]_0^2 - \left[\frac{x^4}{4} - 2x^3 + 4x^2\right]_2^4 = 4 - \cancel{16} + \cancel{16} - 0 - (4^3 - 128 + 64 - 4 + \cancel{16} - \cancel{16})$$
$$= 4 + 4 = \boxed{8}$$

---

20 $y = \ln x$
eje OX
tgte en $x = e$

tangente: $y - y_0 = m(x - x_0)$
$y - f(e) = f'(e)\,(x - e)$

$f'(x) = 1/x$ $\qquad y - \ln e = \frac{1}{e}(x - e)$
$y - 1 = \frac{x}{e} - 1$
$y = x/e$



$$A = \int_0^e (x/e)\,dx - \int_1^e \ln x\,dx = \frac{x^2}{2e}\Big]_0^e - [x\ln x - x]_1^e = \frac{e^2}{2e} - 0 - (e\ln e - e - 0 + 1) =$$

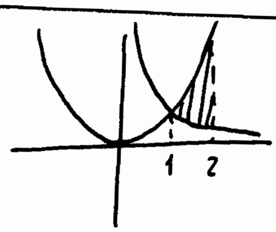
$\int \ln x\,dx = x\ln x - \int \cancel{x}\frac{dx}{\cancel{x}} = x\ln x - x$
$u = \ln x \quad du = \frac{dx}{x}$
$dv = dx \quad v = x$

$= \boxed{\frac{e}{2} - 1}$

---

21 $y = x^2$
$y = \frac{1}{x}$
$x = 2$

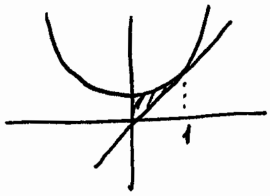
$x^2 = \frac{1}{x}$
$x^3 = 1$
$x = \sqrt[3]{1} = 1$



$$A = \int_1^2 \left(x^2 - \frac{1}{x}\right)dx = \frac{x^3}{3} - \ln x\Big]_1^2 = \frac{8}{3} - \ln 2 - \left(\frac{1}{3} - \ln 1\right) = \frac{8}{3} - \ln 2 - \frac{1}{3} =$$
$$= \boxed{\frac{7}{3} - \ln 2}$$

22 $y = x^2+1$
tgte en $x=1$
oy

$$\left.\begin{array}{l} y' = 2x \\ f'(1) = 2 \\ f(1) = 2 \end{array}\right\} \begin{array}{l} y-2 = 2(x-1) \\ y-2 = 2x-2 \\ y = 2x \end{array}$$

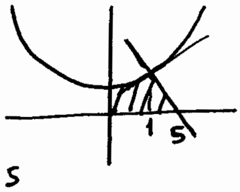


$$A = \int_0^1 \left((x^2+1) - 2x\right) dx = \frac{x^3}{3} + x - x^2 \Big]_0^1 = \frac{1}{3} + 1 - 1 = \boxed{\frac{1}{3}}$$

---

23 $y = x^2+1$
normal en $x=1$
ejes

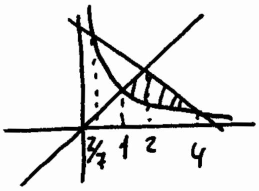
$$y - f(1) = \frac{-1}{f'(1)}(x-1)$$
$$y - 2 = -\frac{1}{2}(x-1)$$
$$y - 2 = -\frac{x}{2} + \frac{1}{2} \qquad -\frac{x}{2} + \frac{5}{2} = 0 \Rightarrow x = 5$$
$$y = -x/2 + \frac{5}{2}$$



$$A = \int_0^1 (x^2+1)\, dx + \int_1^5 \left(-\frac{x}{2} + \frac{5}{2}\right) dx = \frac{x^3}{3} + x \Big]_0^1 + \left(-\frac{x^2}{4} + \frac{5}{2}x\right]_1^5 =$$
$$= \frac{1}{3} + 1 - 0 + \left(-\frac{25}{4} + \frac{25}{2} + \frac{1}{4} - \frac{5}{2}\right) = \boxed{\frac{16}{3}}$$

---

24 $y = x$
$y = \frac{1}{x}$
$y = -\frac{7}{8}x + \frac{15}{4}$



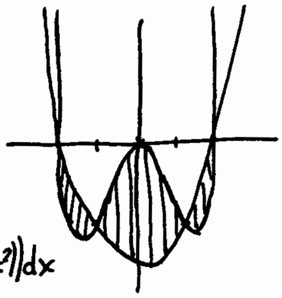
$$x = \frac{1}{x} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$
$$\frac{1}{x} = -\frac{7}{8}x + \frac{15}{4} \Rightarrow 1 = -\frac{7x^2}{8} + \frac{15}{4}x$$
$$7x^2 - 30x + 8 = 0 \left\{\begin{array}{l} x = 4 \\ x = 2/7 \end{array}\right.$$
$$x = -\frac{7}{8}x + \frac{15}{4} \Rightarrow 8x = -7x + 30$$
$$x = 2$$

$$A = \int_1^2 \left(x - \frac{1}{x}\right) dx + \int_2^4 \left(-\frac{7}{8}x + \frac{15}{4} - \frac{1}{x}\right) dx =$$
$$= \frac{x^2}{2} - \ln x \Big]_1^2 + \left(-\frac{7}{8}\frac{x^2}{2} + \frac{15}{4}x - \ln x\right]_2^4 = \frac{4}{2} - \ln 2 - \frac{1}{2} + \cancel{\ln 1} - \frac{7}{8}\frac{16}{2} + \frac{15}{4}4 - \ln 4 + \frac{7}{8}\frac{4}{2} - \frac{15}{4}2 + \ln 2$$
$$= 2 - \cancel{\ln 2} - \frac{1}{2} - 7 + 15 - \ln 4 + \frac{7}{4} - \frac{15}{2} + \cancel{\ln 2} = \boxed{\frac{15}{4} - \ln 4}$$

---

25 $\left.\begin{array}{l} y = x^4 - 4x^2 \\ y = x^2 - 4 \end{array}\right\}$

$$x^4 - 4x^2 = x^2 - 4$$
$$x^4 - 5x^2 + 4 = 0$$
$$t = x^2 \Rightarrow t^2 - 5t + 4 = 0 \left\{\begin{array}{l} t = 1 \Rightarrow x = \sqrt{1} = \pm 1 \\ t = 4 \Rightarrow x = \sqrt{4} = \pm 2 \end{array}\right.$$



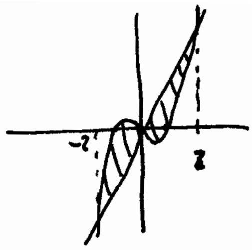
$$A = \int_{-2}^{-1} \left(x^2 - 4 - (x^4 - 4x^2)\right) dx + \int_{-1}^{1} \left(x^4 - 4x^2 - (x^2-4)\right) dx + \int_1^2 \left(x^2 - 4 - (x^4 - 4x^2)\right) dx$$
$$= \int_{-2}^{-1} (5x^2 - x^4 - 4)\, dx + \int_{-1}^{1} (x^4 - 5x^2 + 4)\, dx + \int_1^2 (5x^2 - x^4 - 4)\, dx =$$
$$= \frac{5x^3}{3} - \frac{x^5}{5} - 4x \Big]_{-2}^{-1} + \frac{x^5}{5} - 5\frac{x^3}{3} + 4x \Big]_{-1}^{1} + \frac{5x^3}{3} - \frac{x^5}{5} - 4x \Big]_1^2 =$$
$$= -\frac{5}{3} + \frac{1}{5} + 4 + \frac{40}{3} - \frac{32}{5} - 8 + \left(\frac{1}{5} - \frac{5}{3} + 4 + \frac{1}{5} - \frac{5}{3} + 4\right) + \left(\frac{40}{3} - \frac{32}{5} - 8 - \frac{5}{3} + \frac{1}{5} + 4\right) = \boxed{8}$$

(26) $y = x^3 - x$ $\quad y = 3x$

$x^3 - x = 3x$

$x^3 - 4x = 0$

$x\,(x^2-4) = 0 \begin{cases} x = 0 \\ x = \end{cases}$



$$A = 2\cdot\int_0^2 3x - (x^3 - x)\,dx = 2\cdot\int_0^2 (4x - x^3)\,dx = 2\left[2x^2 - \frac{x^4}{4}\right]_0^2 =$$

$$= 2\,(8 - 4 - 0) = 2\cdot 4 = \boxed{8}$$

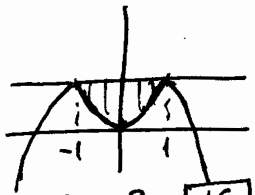
---

(27) $y = -x^4 + 2x^2$ $\quad y = 1$

$-x^4 + 2x^2 = 1$

$-x^4 + 2x^2 - 1 = 0$

$t = x^2 \Rightarrow -t^2 + 2t - 1 = 0 \Rightarrow t = 1 \Rightarrow x = \sqrt{1} = \pm 1$



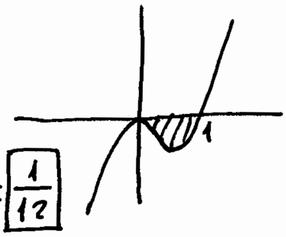
$$A = \int_{-1}^{1} 1 - (-x^4 + 2x^2)\,dx = \left. x + \frac{x^5}{5} - 2\frac{x^3}{3}\right]_{-1}^{1} = 1 + \frac{1}{5} - \frac{2}{3} + 1 + \frac{1}{5} - \frac{2}{3} = \boxed{\frac{16}{15}}$$

---

(28) $y = x^3 - x^2$ $\quad$ eje OX

$x^3 - x^2 = 0$

$x^2(x-1) = 0 \begin{cases} x = 0 \\ x = 1 \end{cases}$



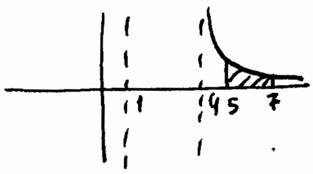
$$A = -\int_0^1 (x^3 - x^2)\,dx = -\left[\frac{x^4}{4} - \frac{x^3}{3}\right]_0^1 = -\frac{1}{4} + \frac{1}{3} = \boxed{\frac{1}{12}}$$

---

(29) $y = \dfrac{x}{x^2 - 5x + 4}$

$x = 5$

$x = 7$



$$A = \int \frac{x}{x^2 - 5x + 4}$$

$$\frac{x}{x^2-5x+4} = \frac{x}{(x-1)(x-4)} = \frac{A}{x-1} + \frac{B}{x-4} = \frac{A(x-4) + B(x-1)}{(x-1)(x-4)} \Rightarrow$$

$x = A(x-4) + B(x-1)$

$x = 1 \Rightarrow 1 = -3A \rightarrow A = -1/3$

$x = 4 \Rightarrow 4 = 3B \rightarrow B = 4/3$

$$A = \int \frac{x}{x^2-5x+4}\,dx = \int \left(\frac{-1/3}{x-1} + \frac{4/3}{x-4}\right) dx = \left. -\frac{1}{3}\ln|x-1| + \frac{4}{3}\ln|x-4| \right] =$$

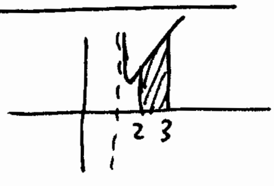
$$= -\frac{1}{3}\ln \;\; + \frac{4}{3}\ln \;\; + \frac{1}{3}\ln \;\; - \frac{4}{3}\ln 1 = \boxed{\frac{4}{3}\ln 3 + \frac{1}{3}\ln 4 - \frac{1}{3}\ln 6}$$

---

(30) $y = \dfrac{x^2}{2x-2}$

$x = 3$

$y = 2$



$$A = \int_2^3 \frac{x^2}{2x-2}\,dx = \frac{1}{2}\int_2^3 \frac{x^2}{x-1} =$$

$$\begin{array}{l|l} x^2 & x-1 \\ \hline -x^2 + x & x+1 \\ \quad x & \\ \quad -x+1 & \\ \quad\quad 1 & \end{array}$$

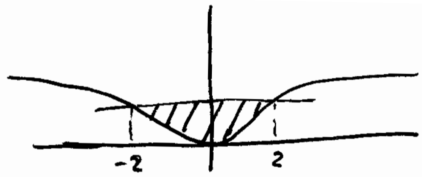
$$= \frac{1}{2}\int_2^3 \left(x + 1 + \frac{1}{x-1}\right) dx = \frac{1}{2}\left[\frac{x^2}{2} + x + \ln|x-1|\right]_2^3 =$$

$$= \frac{1}{2}\left(\frac{9}{2} + 3 + \ln 2 - 2 - 2 - \cancel{\ln 1}\right) = \boxed{\frac{7}{4} + \frac{1}{2}\ln 2}$$

(31) $\left.\begin{array}{l} y=\ln(x^2+1) \\ y=\ln 5 \end{array}\right\}$ $\quad \ln(x^2+1)=\ln 5$

$x^2+1=5$

$x^2=4 \Rightarrow x=\pm 2$



$$A=\int_{-2}^{2}\ln 5-\ln(x^2+1)=\ln 5\,x-\int_{-2}^{2}\ln(x^2+1)$$

$$\int \ln(x^2+1)dx = x\ln(x^2+1)-\int\frac{2x^2}{x^2+1}dx = x.\ln(x^2+1)-2\int\left(1-\frac{1}{x^2+1}\right)dx = x\ln(x^2+1)-2x+2\operatorname{arctg}x$$

$u=\ln(x^2+1) \quad du=\frac{2x}{x^2+1}$

$dv=dx \quad v=x$

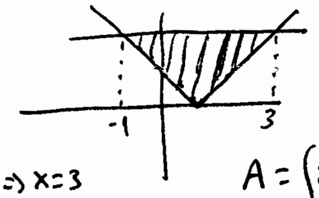
$\begin{array}{l|l} x^2 & x^2+1 \\ -x^2-1 & 1 \\ \hline -1 & \end{array}$

$$\ln 5.x\Big]_{-2}^{2}-\Big[x\ln(x^2+1)-2x+2\operatorname{arctg}x \quad -(2\ln 5-4+2\operatorname{arctg}2+2\ln 5-4-2\operatorname{arctg}(-2))=$$

$$=4\ln 5-(4\ln 5-8+2\operatorname{arctg}2+2\operatorname{arctg}2)= \quad \boxed{8-4\operatorname{arctg}2}$$

---

(32) $\left.\begin{array}{l} y=|x-1| \\ y=2 \end{array}\right\}$



$|x-1|=\begin{cases}-x+1 & x<1 \\ x-1 & x>1\end{cases}$

$|x-1|=2\begin{cases}x-1=2 \Rightarrow x=3 \\ x-1=-2 \Rightarrow x=-1\end{cases}$

$$A=\int_{-1}^{1}2-(-x+1)\,dx+\int_{1}^{3}2-(x-1)\,dx=$$

$$=2x+\frac{x^2}{2}-x\Big]_{-1}^{1}+2x-\frac{x^2}{2}+x\Big]_{1}^{3}=$$

$$=x+\frac{x^2}{2}\Big]_{-1}^{1}+3x-\frac{x^2}{2}\Big]_{1}^{3}=1+\frac{1}{2}+1-\frac{1}{2}+\left(9-\frac{9}{2}-3+\frac{1}{2}\right)=2+6-4=\boxed{4}$$

---

(33) $\left.\begin{array}{l} y=-x^2+2x \\ y=x^3(x-2) \end{array}\right|$ $\quad -x^2+2x=x^4-2x^3$

$x^4-2x^3+x^2-2x=0$

$x(x^3-2x^2+x-2)=0 \left\langle\begin{array}{l}\boxed{x=0} \\ x^3-2x^2+x-2=0 \rightarrow\end{array}\right.$

$\begin{array}{r|rrrr} & 1 & -2 & 1 & -2 \\ 2 & & 2 & 0 & 2 \\ \hline & 1 & 0 & 1 & \boxed{0} \end{array}$

$(x-2)(x^2+1)=0$

$\boxed{x=2}$

$$A=\left|\int_0^2\left(-x^2+2x-(x^4-2x^3)\right)dx\right|=$$

$$=\left|-\frac{x^3}{3}+x^2-\frac{x^5}{5}+2\frac{x^4}{4}\Big]_0^2\right|=\left|-\frac{8}{3}+4-\frac{32}{5}+8-0\right|=\boxed{\frac{44}{15}}$$

---

(34) $\left.\begin{array}{l} y=x^2-2x \\ y=x^3(x-2) \end{array}\right|$ $\quad x^2-2x=x^4-2x^3$

$x^4-2x^3-x^2+2x=0$

$x(x^3-2x^2-x+2)=0 \left\langle\begin{array}{l}x=0 \\ x^3-2x^2-x+2=0 \rightarrow\end{array}\right.$

$\begin{array}{r|rrrr} & 1 & -2 & -1 & 2 \\ 2 & & 2 & 0 & -2 \\ \hline & 1 & 0 & -1 & \boxed{0} \end{array}$

$(x-2)(x^2-1)=0 \begin{cases}x=2 \\ x=\pm 1\end{cases}$

$$A=\left|\int_{-1}^{0}x^4-2x^3-(x^2-2x)\,dx\right|+\left|\int_0^1 x^4-2x^3-(x^2-2x)dx\right|+\left|\int_1^2 x^4-2x^3-(x^2-2x)dx\right|=$$

$$=\left|\frac{x^5}{5}-\frac{x^4}{2}-\frac{x^3}{3}+x^2\Big]_{-1}^{0}\right|+\left|\frac{x^5}{5}-\frac{x^4}{2}-\frac{x^3}{3}+x^2\Big]_0^1\right|+\left|\frac{x^5}{5}-\frac{x^4}{2}-\frac{x^3}{3}+x^2\Big]_1^2\right|=$$

$$=\left|0+\tfrac{1}{5}+\frac{1}{2}-\frac{1}{3}-1\right|+\left|\tfrac{1}{5}-\frac{1}{2}-\frac{1}{3}+1-0\right|+\left|\frac{32}{5}-8-\frac{8}{3}+4-\frac{1}{5}+\frac{1}{2}+\frac{1}{3}-1\right|=\frac{19}{30}+\frac{11}{30}+\frac{19}{30}=\boxed{\frac{49}{30}}$$

35) $y=-x^4+2x^2$
$y=x+2$
$y=-x+2$

$\left.\begin{array}{l} y=-x^4+2x^2 \\ y=x+2 \end{array}\right\}$ $-x^4+2x^2=x+2$
$x^4-2x^2+x+2=0$

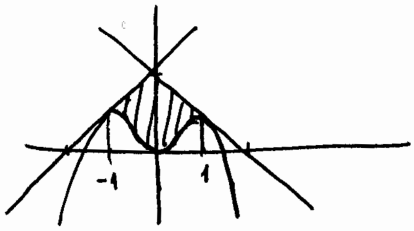
$$\begin{array}{r|rrrrr} & 1 & 0 & -2 & 1 & 2 \\ -1 & & -1 & 1 & 1 & -2 \\ \hline & 1 & -1 & -1 & 2 & \boxed{0} \end{array}$$

$\boxed{x=-1}$

$\left.\begin{array}{l} y=-x^4+2x^2 \\ y=-x+2 \end{array}\right\}$ $-x^4+2x^2=-x+2$
$x^4-2x^2\ x+2=0$

$$\begin{array}{r|rrrrr} & 1 & 0 & -2 & -1 & 2 \\ 1 & & 1 & 1 & -1 & -2 \\ \hline & 1 & 1 & -1 & -2 & \boxed{0} \end{array}$$

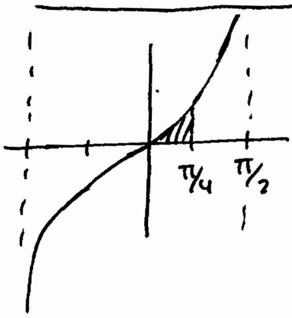
$\boxed{x=1}$



$$A=\int_{-1}^{0} x+2-(-x^4+2x^2)\,dx+\int_0^1 -x+2-(-x^4+2x^2)\,dx=$$
$$=\frac{x^2}{2}+2x+\frac{x^5}{5}-2\frac{x^3}{3}\Big]_{-1}^{0}+-\frac{x^2}{2}+2x+\frac{x^5}{5}-2\frac{x^3}{3}\Big]_0^1=$$
$$=0-\left(\frac{1}{2}-2-\frac{1}{5}+\frac{2}{3}\right)+\left(-\frac{1}{2}+2+\frac{1}{5}-\frac{2}{3}-0\right)=\frac{31}{30}+\frac{31}{30}=\boxed{\frac{31}{15}}$$

---

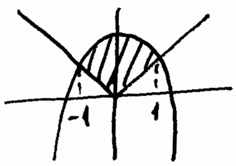
36) $y=tgx$
OX
$x=\pi/4$



$$A=\int_0^{\pi/2} tgx\,dx=\int_0^{\pi/2}\frac{senx}{cosx}dx=-\int_0^{\pi/2}\frac{-senx}{cosx}dx=$$
$$=-\ln|cosx|\Big]_0^{\pi/4}=-\ln|\cos\pi/4|+\ln|\cos\ |=$$
$$=-\ln|\sqrt{2}/2|+\ln 1=\ln\left|\frac{\sqrt{2}}{2}\right|^{-1}=\ln\left|\frac{2}{\sqrt{2}}\right|=\boxed{\ln\sqrt{2}}$$

---

37) $\left.\begin{array}{l} y=2-x^2 \\ y=|x| \end{array}\right\}$



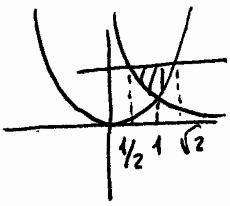
$2-x^2=x$
$x^2+x-2=0$
$\begin{cases} x=1 \\ x=-2 \end{cases}$

$2-x^2=-x$
$x^2-x-2=0$
$\begin{cases} x=-1 \\ x=+2 \end{cases}$

$$A=\int_{-1}^{0}(2-x^2-(-x))dx+\int_0^1(2-x^2-x)dx=\int_{-1}^{0}(2-x^2+x)dx+\int_0^1(2-x^2-x)\,dx=$$
$$=2x-\frac{x^3}{3}+\frac{x^2}{2}\Big]_{-1}^{0}+\ 2x-\frac{x^3}{3}-\frac{x^2}{2}\Big]_0^1=0-\left(-2+\frac{1}{3}+\frac{1}{2}\right)+\ 2-\frac{1}{3}-\frac{1}{2}-0=$$
$$=2-\frac{1}{3}-\frac{1}{2}+2-\frac{1}{3}-\frac{1}{2}=4-\frac{2}{3}-1=\boxed{7/3}$$

---

38) $y=x^2$
$y=\frac{1}{x}$
$y=2$



$x^2=\frac{1}{x}$
$x^3=1$
$x=\sqrt[3]{1}$
$x=1$

$\frac{1}{x}=2$
$x=1/2$

$x^2=2$
$x=\pm\sqrt{2}$

$$A=\int_{1/2}^{1}\left(2-\frac{1}{x}\right)dx+\int_1^{\sqrt{2}}(2-x^2)dx=2x-\ln x\Big]_{1/2}^{1}+\ 2x-\frac{x^3}{3}\Big]_1^{\sqrt{2}}=$$
$$=2-\cancel{\ln 1}-2\cdot 1/2+\ln 1/2+2\sqrt{2}-\frac{\sqrt{2}^3}{3}-2+\frac{1}{3}=\cancel{2}-1-\ln 2+2\sqrt{2}-\frac{2\sqrt{2}}{3}-\cancel{2}+1/3=$$
$$=\boxed{\frac{4}{3}\sqrt{2}-\frac{2}{3}-\ln 2}$$